SOS HGS Sanothimi Bhaktapur

Convergence of light exactly at focus by parabolic mirror

Without spherical aberration

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Foreword

During my first year of high school studies, in a class of physics, in SOS HGS my geometric optics teacher told me that a spherical mirror wouldn't completely converge all the rays of light falling parallel on it essentially on the focus, it would cause spherical aberration – inability to converge all the parallel rays to focus. He added we require a parabolic mirror to remove this aberration.

I immediately got lost from the further discussion of the class which was based on the belief ; rather crude approximation that the spherical mirror would essentially converge all the light falling to it at the focus. I needed exact physical situation (during those early stubborn days of my physics class I hated approximation; I needed everything according to theory). I sat with pen and paper to find out how the spherical mirror would converge all the rays of light at the focus. But I knew nothing more than a equation of the parabola. A general equation of second degree $y = ax^2 + bx + c$. My bench partner suggested me to go with a simpler equation of parabola $y = x^2$. I refused because I hated special cases; I required general solutions to every equation. (Even my teacher in the class was tired of my quest of general approach to all the problems. Not just my physics teacher even chemistry and maths teacher probably hated me for my rigorous questions on general approach). I was not to get to the solution. My bench partner even called me crazy (I probably had been so). Never did I get the solution to the problem nor did I gave hope of getting one. I could have searched on the internet or consulted teacher to help me find it. But my passion was not to get the answer but to find how can I utilize the chemical reactions going on in my brain to find out the solution.

During first year of my high school we did not have to deal with much calculus and in those days we knew almost nothing about calculus nevertheless I knew to differentiate simple functions (without knowing what differentiation meant).

Whenever I saw a motorbike or a bus with a convex mirror on it I got stuck on the solution to the problem.

After nearly an year of it during my second year in high school I got my original solution (it may coincide with what other have done; but its my own). I was so happy; I got the solution with assistance of the chemical reactions going on within my own brain and that was enjoyable without a doubt.

The solution is based on the principle that the angle of incidence is equal to the angle of reflection. Few days after that I found one solution to this problem in Feynman's lecture Vol -1 based on Fermant's principle of shortest time. The proof was so elegant; but I never regretted for the year I spent in finding to the solution of the problem using the chemical reactions goin in my brain.
Here I present the same proof. It is rather crude; but I love it; I have my origanility presented here.
Comments and suggestions are enjoyable and are always welcome.

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Sol[n]

Let a parabolic mirror be placed parallel to X axis; the equation of which is $y^2 = 4ax$. Let a general point $A(at^2, 2at)$ on the parabola.

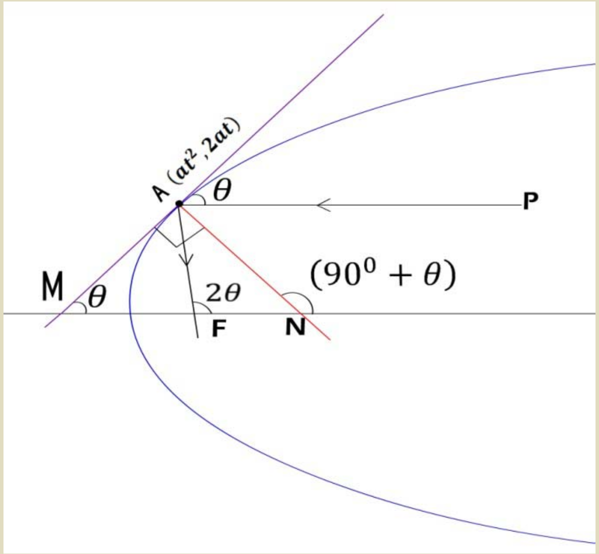
Here the equation of parabola is
$y^2 = 4ax$

Differentiation both sides w.r.t. x:

$\frac{d}{dx}(y^2) = \frac{d}{dx}(4ax)$

Or, $2y\frac{dy}{dx} = 4a$

Or, $\frac{dy}{dx} = \frac{2a}{y}$



Since the first derivative of the function gives the slope of the tangent at any point on the function; the slope of any tangent through the point (x,y) on the function is $(m) = \frac{2a}{y}$

So clearly the slope of tangent through $(at^2, 2at)$ whose y co-ordinate is $2at$ is

$$m = \frac{2a}{2at} = \frac{1}{t}$$

Let the tangent makes angle θ with the x axis
So, slope= $\tan\theta = m = \frac{1}{t}$

Now,
Let a ray of light PA parallel to principal axis strike the mirror at point A and gets reflected along AF making angle of incidence equal to angle of reflection. Let us draw a perpendicular AN at the point A

In the figure we can show that:

$\measuredangle PAN = 90^0 - \theta$

$\measuredangle NAF = \measuredangle PAN$ (Angle of incidence equal to angle of reflection)

$\measuredangle NAF = 90^0 - \theta$

Again,

$\measuredangle ANX = \measuredangle AMN + \measuredangle MAN$ (Exterior angle of triangle equal to sum of interior angle)

$\measuredangle ANX = 90^0 + \theta$

Also,
$\measuredangle NAF + \measuredangle \text{AF}N = \measuredangle \text{A}NX$
$\measuredangle \text{AF}N = \measuredangle \text{A}NX - \measuredangle NAF$
$\measuredangle \text{AF}N = (90^0 + \theta) - (90^0 - \theta)$
$\measuredangle \text{AF}N = 2\theta$

Now,
Slope of the reflected ray m_r is given by
$m_r = \tan(\measuredangle AFN)$
$m_r = \tan(2\theta)$
$$m_r = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Substuting the value of $\tan\theta$

$$m_r = \frac{2 \cdot \frac{1}{t}}{1 - \frac{1}{t^2}}$$

$$m_r = \frac{2 \cdot \frac{1}{t} \cdot t^2}{t^2 - 1}$$

$$m_r = \frac{2t}{t^2 - 1}$$

Now the equation of line AF (reflected ray) is
$y - 2at = m_r(x - at^2)$

Substuting the value of m_r we get;
$$y - 2at = \left(\frac{2t}{t^2 - 1}\right)(x - at^2)$$

This is the equation of the reflected ray after reflection on any point on parabola.

Now we need to show that the reflected ray always passes through the focus of the parabola. That is to say the co ordinate of focus of parabola should satisfy the equation of the reflected ray.

We know that the co ordinate of focus of parabola $y^2 = 4ax$ is $(a, 0)$. So the point $(a, 0)$ should satisfy the equation of reflected line(ray).

On substation of (x,y) with $(a, 0)$ we get:

$$0 - 2at = \left(\frac{2t}{t^2-1}\right)(a - at^2)$$

$$\frac{0 - 2at}{(a - at^2)} = \left(\frac{2t}{t^2 - 1}\right)$$

$$\frac{-2at}{-a(t^2 - 1)} = \left(\frac{2t}{t^2 - 1}\right)$$

$$\left(\frac{2t}{t^2 - 1}\right) = \left(\frac{2t}{t^2 - 1}\right)$$

Which is true; so the reflected ray always passes through the focus of the parabola.

Isn't that great???????

So a parabolic mirror removes the chromatic aberration of caustic curve caused by the spherical mirror.

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